

$$1. \quad x e^{x^2} dx + (y^5 - 1) dy = 0$$

$$y(0) = 0$$

$$\text{für } u = x^2$$

$$\frac{du}{dx} = 2x \Rightarrow 2x dx = du \Rightarrow x dx = \frac{1}{2} du$$

$$\text{also} \quad \text{mit} \quad x e^{x^2} dx = e^{x^2} x dx = e^u \frac{1}{2} du$$

$$\frac{1}{2} e^u du + y^5 dy - dy = 0$$

$$\int \frac{1}{2} e^u du + \int y^5 dy - \int dy = c_1$$

$$\frac{1}{2} e^u + \frac{1}{6} y^6 - y = c_1$$

$$\text{mit } u = x^2$$

$$\frac{1}{2} e^{x^2} + \frac{1}{6} y^6 - y = c_1$$

$$\text{bei } x=0, y=0$$

$$\frac{1}{2} e^{0^2} + \frac{1}{6} 0^6 - 0 = c_1$$

$$c_1 = \frac{1}{2}$$

$$\text{also} \quad \frac{y^6}{6} - y + \frac{1}{2} e^{x^2} = \frac{1}{2}$$